Motion of Particulate Material Ejected from a Rotating Space Platform

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Nomenclature

$g_{\oplus}$ = gravity at the Earth's surface
$\underline{\underline{L}}_{BA}$ = transformation matrix from attitude coordinates to body-fixed coordinates
$R_{\oplus}$ = radius of the Earth
$\boldsymbol{R}_p$ = position vector from platform center of mass to particle
$\boldsymbol{r}_p$ = geocentric position vector of particle p
$\boldsymbol{r}_s$ = geocentric position vector to platform
ω_A = angular velocity of attitude coordinate system
$\omega_{B/A}$ = angular velocity of platform relative to attitude coordinate system
$(\ )^A$ = vector quantity expressed in attitude coordinate components
$(\ )^B$ = vector quantity expressed in body-fixed coordinate components
$(\dot{\ })$ = time rate of change of the components of the vector quantity ( )

Abstract

THE dynamics of particulate material ejected from a rotating space platform are investigated. A cloud of the material is modeled as a number of discrete finite-sized particles so that the problem of tracking the material can be solved by simulating the motion of nonmutually interacting two-body systems. The equations of motion, which form the basis of the simulation, are written with respect to a coordinate system attached to a rotating space platform in Earth orbit and include the effects of gravity and other *external* forces. The resulting nonlinear equations are integrated numerically to obtain time histories of each particle's position and velocity. The time histories indicate that for certain initial conditions, particularly those corresponding to altitudes where aerodynamic drag is significant, some of the ejected particulate material may recontact the platform.

Contents

Particulate matter can be ejected from a spacecraft with a wide range of velocities, say, from 10^{-2} m/s for outgassing to 10^4 m/s for rocket plumes. Large space platforms require significant amounts of propellant for maneuvers such as "slewing" and/or stationkeeping. In particular, if rapid slewing of a large platform ($\simeq 10^6$ kg) is required, the quantity of exhaust gas, or particulate matter, from the propulsion system may be considerable, perhaps on the order of 10^3 kg/s.[1] In addition, some projected platform applications[2] may require the use of open-cycle turbines, wherein a large amount of the working fluid is dumped overboard. Consideration must be given to the direction in which particulate material is exhausted, and at least a general knowledge of the behavior of the material is needed. The prediction problem is complicated by the fact that external forces, such as aerodynamic drag, may act on the ejected matter. These forces tend to change the path of the matter from that which classical Keplerian theory predicts.

The present work represents an extension of a procedure proposed by Lee[3] to include the effects of platform rotation in the dynamic equations of motion. The particulate material may be thought of as single particles, or as a conglomeration (cloud) of many particles. Only the ordered motion of such material is analyzed. Particle-particle interactions such as backscattering effects are not modeled. The satellite, or platform, from which the particles are initially released can be in a circular, or an elliptical orbit, or may be moving along any other trajectory that can be described analytically. The effects of aerodynamic drag are included.

It should be noted that no effort is made here to model controlled platform motion as in Ref. 1. The platform is assumed to be a rigid body undergoing uniform rotation about an axis of maximum inertia. Three dextral, orthogonal coordinate systems are used to describe relative particle motion. Figure 1a shows the Earth-centered coordinate system *EXYZ*, which is considered to be fixed in an inertial reference frame. The orientation of the system is defined by requiring that the Z axis pass through the north pole and the X axis lie along the prime meridian. Also shown is the "attitude-fixed" coordinate system that has its origin at the orbiting platform's center of mass, but is oriented so that the x_a axis is always tangent to the platform's velocity vector, and the z_a axis points toward the center of the Earth (if the orbit is circular). Thus, the orbital plane of the platform's center of mass always coincides with the $x_a z_a$ plane.

Since the problem is formulated so that effects due to the platform's rotational motion can be included, a coordinate system attached to the body is also defined. Figure 1b shows the "body-fixed" system $Ox_b y_b z_b$. This system, also with its origin at the body's center of mass, is rotated through the angles θ_3, θ_2, and θ_1 from the attitude system in the standard flight dynamics manner (see Etkin[4]). The three angles represent yaw, pitch, and roll angles, respectively, and can be used to define any arbitrary orientation of the body. This particular system is used because of its usefulness in visualizing platform motion.

Upon performing the necessary coordinate transformations, the equation governing the motion of a particle relative to the platform may be written as

$$\begin{aligned}
\underline{\underline{L}}_{AB}\ddot{\boldsymbol{R}}_p^B = & -g_{\oplus} R_{\oplus}^2/r_p^3\, \boldsymbol{r}_p^A - \ddot{\boldsymbol{r}}_s^A - 2\tilde{\omega}_A^A \dot{\boldsymbol{r}}_s^A + \boldsymbol{a}_{p_A}^A \\
& - 2\,(\tilde{\omega}_A^A + \underline{\underline{L}}_{AB}\, \tilde{\omega}_{B/A}^B\, \underline{\underline{L}}_{BA})\, \underline{\underline{L}}_{AB}\, \dot{\boldsymbol{R}}_p^B \\
& - \dot{\tilde{\omega}}_A^A\, (\boldsymbol{r}_s^A + \underline{\underline{L}}_{AB}\, \boldsymbol{R}_p^B) - \underline{\underline{L}}_{AB}\, \dot{\tilde{\omega}}_{B/A}^B\, \boldsymbol{R}_p^B \\
& - \tilde{\omega}_A^A[\tilde{\omega}_A^A(\boldsymbol{r}_s^A + \underline{\underline{L}}_{AB}\, \boldsymbol{R}_p^B)] \\
& - \underline{\underline{L}}_{AB}\tilde{\omega}_{B/A}^B(\tilde{\omega}_{B/A}^B\, \boldsymbol{R}_p^B) - 2\,\tilde{\omega}_A^A\,(\underline{\underline{L}}_{AB}\tilde{\omega}_{B/A}^B\, \boldsymbol{R}_p^B)
\end{aligned}$$

where the first term on the right-hand side represents gravita-

Received March 22, 1988; revision received Aug. 25, 1988.

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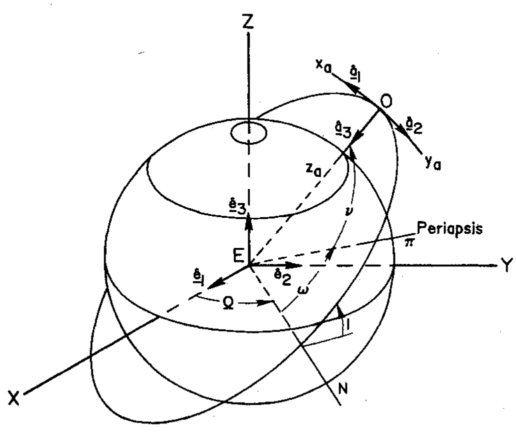
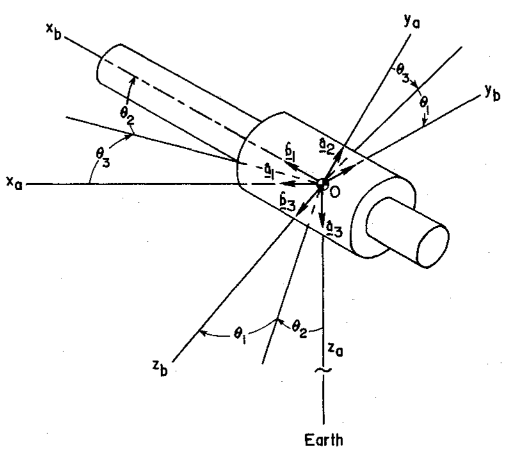


a) **Inertial and attitude fixed**



b) **Attitude-fixed and body-fixed**

Fig. 1 Inertial, attitude-fixed, and body-fixed coordinate systems.

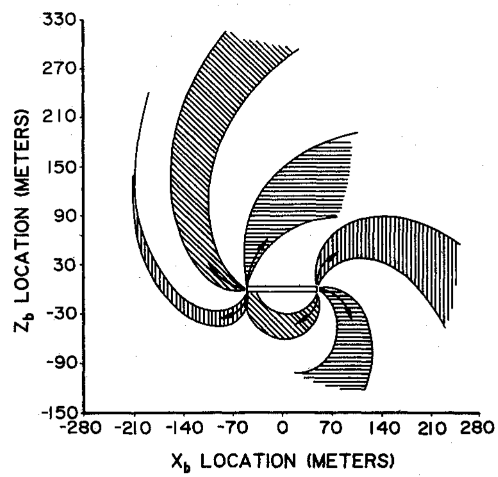


Fig. 2 Cloud formation around a rotating platform (initial platform angle = 0.0 deg), body-fixed coordinate system.

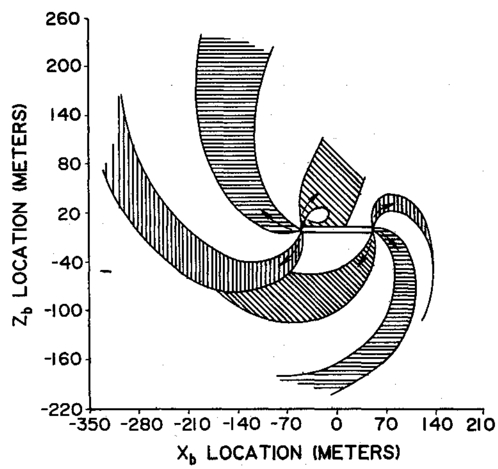


Fig. 3 Cloud formation around a rotating platform (initial platform angle = 90.0 deg), body-fixed coordinate system.

tional acceleration, $a_{p_A}^A$ is the acceleration due to aerodynamic drag, and $\underline{\underline{L}}_{AB}$ is the transpose of $\underline{\underline{L}}_{BA}$.

Results

An exhaust cloud can be simulated by tracking a family of particles ejected simultaneously (or otherwise) with relative velocities of identical magnitudes but different directions. Such "clouds" are shown in Fig. 2. Each consists of a fan-shaped distribution of particles ejected, as shown, with half angles of 20 deg, each with a relative velocity magnitude of 30 m/s from a platform of 100 m length rotating about its y axis with an angular velocity of -0.35 rad/s and moving in a 250-km altitude circular orbit. At this altitude, aerodynamic drag is significant. The plot is of the body-fixed coordinates of particles. The body-fixed coordinate system is initially aligned with the attitude coordinate system, but is, of course, rotating relative to it. Note that, for these conditions, it is possible for the platform to rotate through the cloud. Figure 3 illustrates the case where the platform is initially rotated 90 deg about its y axis relative to the attitude coordinate system. Note that the canceling effect of rotation allows drag forces to quickly blow the cloud back toward the platform so that many particles recontact it.

References

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